An Application of Pulley-Cable Element in Solving Form Finding Problem for Cable-Supported Structures

Dang Dang Tung
Faculty of Civil Engineering, HoChiMinh city University of Technology, HCMC, Vietnam

Nguyen Tang Thanh Binh
Faculty of Civil Engineering, HoChiMinh city University of Technology, HCMC, Vietnam

Abstract: The realization of desired cable - supported structures depends on form finding procedures by its geometrical nonlinearity. The aim of this study is that, even if a special calculation technique for finding form is not used, applying pulley - cable element can solve the form finding problem for cable - supported structures. That means form finding and structural analysis for cable - supported structures could be done with the same technique. In this paper, some functions are also appended for form finding of cable truss, cable net and suspension bridge. This assumption can be represented if cable passes through pulley on supports, other cables or beam element. The pulley - cable element has been developed in this study is formulated by modified functional of variational principle and has the same feature as ordinary finite element based on the displacement method in which displacements are unknown variables. Some numerical examples are presented to show the accuracy and efficiency of the proposed form finding method.

Keywords: Cable element, pulley, form finding, pre-stress.

1 INTRODUCTION

Due to their esthetic appearance as well as efficient utilization of structural material, long-span cable-supported bridges have gained much popularity in recent decades. On the other hand, problem of form finding of cable-supported structures becomes very important since cable element is flexible and shape of cable does not settle until acting own weight, pre-stress and external loads.

Until a load acts, cable element has greatly different property that the configuration is not found with cable structure, in comparison with other structural elements. Therefore, more accurate and precise analysis techniques based on usual displacement method are required to consider the various nonlinear behaviors of cables.

In principle, configuration of a cable structure is known as a solution of equilibrium equation. However, general cable structure necessitates the "shape finding" procedure in its erection: to fulfill the design requirements, the equilibrium state needs to settle in variation of structural parameters such as the cable length and nodal coordinates.

Many researchers have researched on shape finding problem till now. However, after settling the configuration of structure, it is necessary to have to use another technique of structural analysis to solve the problem usually. In other words, shape-finding method is not the same as structural analysis method to solve the problem. In this study, by using cable element with pulley, even if a special calculation technique for finding form is not used, the shape finding problem can be solved. That means shape finding method and structure analysis method can be done with the same technique. It is main characteristic of this study.

There are researches of McDonald & Peyrot (1988) and Aufaure (1993) in analysis of cable structure with pulley. McDonald & Peyrot (1988) derived a "pulley element" from elastic catenary cable element. He pointed out that multiple equilibrium configurations exist in sagged cable system with pulley. However, because the analytical solution under the one direction uniform like own weight of the cable is being used in this paper, application to other arbitrary loading condition is difficult to solve. Aufaure (1993) presented a "pulley element" of three-node cable element in which contains pulley and adjacent cable. This cable element with pulley is not used for cable subjected own weight because of adjoining cables through a pulley are treated by straight cable.

There are various conditions of form finding for cable structures. The condition of equal axial force of cables is considered in this paper. This situation can be represented if the cable passes through the pulley, other cables or other beam element.

Therefore, a cable element is proposed in this paper for flexible cable structures with pulleys. This element is formulated by modified functional of variational principle. It has the same features as ordinary finite element based on the displacement method in which displacements are unknown variables.

Furthermore, this paper presents a method to determine shape finding for suspended structures based on introducing some constraint conditions of cable pre-stress. First, known horizontal component of unit vector of cable axial force is assumed to be equal at all pulleys. Secondly, displacement is supposed to be equal zero at the position in which the deck is supported by cable.

Above-mentioned conditions are reasonable because it is considered bending moment of tower does not happen under the action of the dead load of deck and bending moment of the deck has the same features as bending moment of continuous beam. Detailed procedures to analyze the configuration of cable truss system and suspension bridge are presented to show the effectiveness of proposed shape finding method.

In process of induction of the proposed cable element, some assumptions are made as follows: the stress-strain relationship of all materials always remains within linear elastic range during the whole nonlinear computation, the cross-section area of elements remains unchanged during deformation, the cable element is assumed to be perfectly flexible and to possess only tension stiffness, and it is incapable of resisting compression, shear and bending forces. Friction between pulley and cable can be considered too small and to be neglected.

2 CABLE ELEMENT FORMULATION

The foundational formulation of the cable element that it was induced in Iwasaki & Nagai (2002) is shown here.

2.1 Functional of cable element

Suppose that we can get a solution satisfied equilibrium condition and compatibility condition in a certain incremental step. By using incremental displacement vectors $\Delta \boldsymbol{u}_a, \Delta \boldsymbol{u}_b$ at both ends of the cable element and incremental axial force vector $\Delta \boldsymbol{N}_c^*$ at the center of the cable element as independent variations, until the next incremental step, the functional of total incremental potential energy of the cable element can be expressed as follows.

$$\Pi_C = \int_0^l \left\{ -\frac{\Delta N^2}{2EA} + \left(F_x^* - N^*\right) \right\} dx + \left[\left(\mathbf{N}^* - n_x \mathbf{Q}^*\right)^T \Delta \mathbf{u} \right]_0^l \quad (1)$$

where, $\boldsymbol{Q}^*$ is the vector of concentrated load at the end of cable element in the next incremental step; $\boldsymbol{N}^*$ is the element internal axial force vector in the next incremental step; n_x is the direction cosine between the outward normal of the cross section at the end of the cable element and the x direction. The value of n_x is -1 at $x=0$, and $+1$ at $x=l$. ΔN is incremental quantity of the axial force, and EA is extension rigidity.

In Equation 1, F_x^* and N^* are given by the element internal axial force vector $\boldsymbol{N}^*$ and are indicated:

$$F_x^* = \boldsymbol{e}^T \boldsymbol{N}^*, \; N^* = \left|\boldsymbol{N}^*\right|, \; \Delta N = N^* - N \quad (2)$$

where, $\boldsymbol{e}$ is the unit tangental vector of the cable at former incremental step, and $\boldsymbol{e}^*$ is the unit tangental vector in the next increment step, it is given by $\boldsymbol{N}^* / N^*$.

$\boldsymbol{N}^*$ in Equation 1 must satisfy equilibrium equation in the cable element. Therefore, $\boldsymbol{N}^*$ indicate by using the axial force vector $\boldsymbol{N}_c^*$ and the vector of uniformly distributed load $\boldsymbol{q}^*$, we have.

$$\boldsymbol{N}^* = \boldsymbol{N}_c^* - \int_{l/2}^{x} \boldsymbol{q}^* dx \quad (3)$$

2.2 Stationary conditions for the cable element functional

The stationary conditions for the functional of the total variation.

$$\int_0^l \left\{ \boldsymbol{e} - \left(1 + \frac{\Delta N}{EA}\right) \boldsymbol{e}^* \right\} dx + \left[\Delta \boldsymbol{u}\right]_0^l = \boldsymbol{0} \quad (4a)$$

$$-\boldsymbol{N}_a^* - \boldsymbol{Q}_a^* = \boldsymbol{0} \quad (4b)$$

$$\boldsymbol{N}_b^* - \boldsymbol{Q}_b^* = \boldsymbol{0} \quad (4c)$$

where, $\boldsymbol{N}_a^*, \boldsymbol{Q}_a^*$ and $\boldsymbol{N}_b^*, \boldsymbol{Q}_b^*$ are values of $\boldsymbol{N}^*$ and $\boldsymbol{Q}^*$ at the end a *(x=0)* and end b *(x=l)* of the cable element respectively. The term of $\int_0^l \boldsymbol{e} dx$ is obtained in Equation 4a indicates position vector from the end a and the end b in the former incremental step. Similarly, the term of $\int_0^l \left(1 + \Delta N / EA\right) \boldsymbol{e}^* dx$ indicates position vector from the end a and the end b in the next incremental step. It is found that the Equation 4a shows compatibility condition of the cable before and after the incremental displacement. And, the second term and the third term of upper equations show the balance of forces at both ends of the cable element.

2.3 Solution method of the cable element functional

Linearization form of Equation 4 with respect to incremental quantity is presented as follow.

$$\begin{pmatrix} -\boldsymbol{H} & -\boldsymbol{I} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix} \begin{Bmatrix} \Delta N_c \\ \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{u}_b \end{Bmatrix} = \Delta\lambda \begin{Bmatrix} -\overline{\boldsymbol{u}}_c \\ \overline{\boldsymbol{p}}_a \\ \overline{\boldsymbol{p}}_b \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{Q}_a + \boldsymbol{N}_a \\ \boldsymbol{Q}_b + \boldsymbol{N}_b \end{Bmatrix} \tag{5}$$

where, $\boldsymbol{Q}$ and $\boldsymbol{q}$ are indicated by the products of criteria load vectors $\overline{\boldsymbol{Q}}$ and $\overline{\boldsymbol{q}}$ with load magnification $\Delta\lambda$ which is presented in Iwasaki et al (2002). And,

$$\overline{\boldsymbol{u}}_c = \int_0^l \left(\frac{\boldsymbol{e}\boldsymbol{e}^T}{EA} + \frac{\boldsymbol{I} - \boldsymbol{e}\boldsymbol{e}^T}{N} \right) \int_{l/2}^x \overline{\boldsymbol{q}}\, dx dx \tag{6a}$$

$$\overline{\boldsymbol{p}}_a = \overline{\boldsymbol{Q}}_a + \int_0^{l/2} \overline{\boldsymbol{q}}\, dx \tag{6b}$$

$$\overline{\boldsymbol{p}}_b = \overline{\boldsymbol{Q}}_b + \int_{l/2}^{l} \overline{\boldsymbol{q}}\, dx \tag{6c}$$

$$\boldsymbol{H} = \int_0^l \left(\frac{\boldsymbol{e}\boldsymbol{e}^T}{EA} + \frac{\boldsymbol{I} - \boldsymbol{e}\boldsymbol{e}^T}{N} \right) dx \tag{6d}$$

where, notify that the axial force is considered not to be equal zero in this process.

The solution that is obtained from Equation 5 for all cable elements, contains error that appears in the process of linearization with respect to increment quantity. It is necessary to find exact solution by iterative computation.

Consider $N_{c(0)}$, $\Delta \boldsymbol{u}_{a(0)}$ and $\Delta \boldsymbol{u}_{b(0)}$ as initial solutions. By using Newton's iterative method, iterative equation of Equation 4 is indicated as follows.

$$\begin{pmatrix} -\boldsymbol{H}_{(k)} & -\boldsymbol{I} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix} \begin{Bmatrix} \Delta \hat{N}_c \\ \Delta \hat{\boldsymbol{u}}_a \\ \Delta \hat{\boldsymbol{u}}_b \end{Bmatrix} = \Delta\hat{\lambda} \begin{Bmatrix} -\overline{\boldsymbol{u}}_{c(k)} \\ \overline{\boldsymbol{p}}_a \\ \overline{\boldsymbol{p}}_b \end{Bmatrix} + \begin{Bmatrix} -\Delta \boldsymbol{v}_{c(k)} \\ \boldsymbol{Q}^*_{a(k)} + \boldsymbol{N}^*_{a(k)} \\ \boldsymbol{Q}^*_{b(k)} - \boldsymbol{N}^*_{b(k)} \end{Bmatrix} \quad (k \geq 0) \tag{7}$$

Where,

$$\overline{\boldsymbol{u}}_{c(k)} = \int_0^l \boldsymbol{h}_{(k)} \int_{l/2}^x \overline{\boldsymbol{q}}\, dx dx \tag{8a}$$

$$\Delta \boldsymbol{v}_{c(k)} = \int_0^l \left\{ \boldsymbol{e} - \left(1 + \frac{\Delta N_{(k)}}{EA} \right) \boldsymbol{e}^*_{(k)} \right\} dx + \left[\Delta \boldsymbol{u}_{(k)} \right]_0^l \tag{8b}$$

$$\boldsymbol{H}_{(k)} = \int_0^l \boldsymbol{h}_{(k)} dx \tag{8c}$$

$$\boldsymbol{h}_{(k)} = \frac{\boldsymbol{e}^*_{(k)} \boldsymbol{e}^{*T}_{(k)}}{EA} + \left(1 + \frac{\Delta N_{(k)}}{EA} \right) \frac{\boldsymbol{I} - \boldsymbol{e}^*_{(k)} \boldsymbol{e}^{*T}_{(k)}}{N^*_{(k)}} \tag{8d}$$

and then,

$$\Delta \boldsymbol{N}_{c(k+1)} = \Delta \boldsymbol{N}_{c(k)} + \Delta \hat{\boldsymbol{N}}_c \tag{9a}$$

$$\Delta \boldsymbol{u}_{a(k+1)} = \Delta \boldsymbol{u}_{a(k)} + \Delta \hat{\boldsymbol{u}}_a \tag{9b}$$

$$\Delta \boldsymbol{u}_{b(k+1)} = \Delta \boldsymbol{u}_{b(k)} + \Delta \hat{\boldsymbol{u}}_b \tag{9c}$$

$\Delta \boldsymbol{N}_c$ is independent variable to each cable element and it can be eliminated before constructing the equation of global structure.

3 CABLE ELEMENT WITH PULLEY

3.1 A finite element of cable passing through pulley where component of unit vector of axial force is assumed to become equal

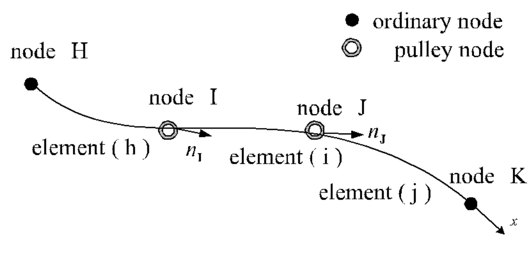

Figure 1. Component of unit vector of axial force is assumed to be equal at pulley.

Consider an assembly of cable (h), (i) and (j) passing through pulleys are installed on nodes I, J shown in Figure 1. It is supposed that in each pulley where the component of unit vector of the axial force is assumed to equal. When this assumption is included into the cable element functional, the functional of cable passing through pulley is expressed as follows.

$$\Pi_C = \Pi_C^{(h)} + \Pi_C^{(i)} + \Pi_C^{(j)} + \Delta\tilde{u}_I \boldsymbol{n}_I^T \left\{ N_I^{*(h)} - N_I^{*(i)} \right\} + \Delta\tilde{u}_J \boldsymbol{n}_J^T \left\{ N_J^{*(i)} - N_J^{*(j)} \right\} \tag{10}$$

where, $\Delta\tilde{u}_I$ and $\Delta\tilde{u}_J$ are Lagrange multipliers. When the functional is stationary for variations of displacement, these constants express sliding displacements of the cable passing through pulley at nodes I, J respectively. These displacements are considered positive if they are laid on the x-axial direction along the cable. Subscript of $\boldsymbol{N}_I^{*(h)}$ denotes the axial force of the cable (h) at node I. From above, generally, in the case of pulley is installed at both ends of cable, the functional of cable element passing through pulley is rewritten as follows.

$$\tilde{\Pi}_C = \Pi_C + \left[\Delta\tilde{u}\boldsymbol{n}^T N^* \right]_0^l \tag{11}$$

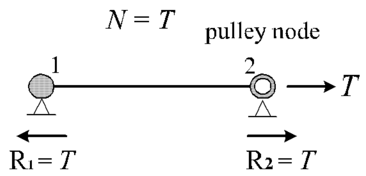

Figure 3. Cable element is considered to fix at both ends.

3.2 Assuming a applied force acts on cable in pulley part

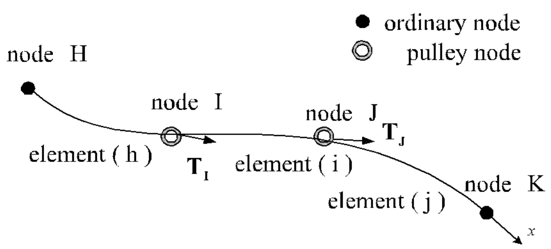

Figure 2. Force acts on the cable in pulley part.

Consider about an assembly of cable (h), (i) and (j) passing through pulleys are installed at nodes I, J as shown in Figure 2. Consider a case that sliding displacements at pulley parts I and J occur under the action of applied forces $T_I + \Delta T_I$ and $T_J + \Delta T_J$. The functional describes this state is expressed as:

$$\tilde{\Pi}_C = \Pi_C + \left[\Delta\tilde{u}\left(N^* - n_x T^* \right) \right]_0^l \tag{12}$$

Pay attention to following two cases in order to understand above assumed content well.

Case 1: The pulley at node 2 is considered to fix in Figure 3. When the force T acts on the cable at node 2, we have.

$$\boldsymbol{N} - \boldsymbol{R}_2 = \boldsymbol{0} \tag{13a}$$

$$N - T = 0 \tag{13b}$$

Reaction force at node 2 is calculated from above equation.

$$\boldsymbol{R}_2 = T\ \boldsymbol{e} \tag{14}$$

Case 2: Both ends of cable element are connected to beam element shown in Figure 4. When the force T acts on the cable in pulley part at node 2, the equilibrium equation of the cable at node 2 becomes.

$$\boldsymbol{N}_c + \boldsymbol{N}_b = \boldsymbol{0} \tag{15a}$$

$$N_c - T = 0 \tag{15b}$$

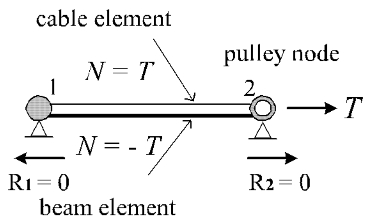

Figure 4. Both ends of cable element are fixed at beam element.

where, $\boldsymbol{N}_c$ and $\boldsymbol{N}_b$ are vectors of the cross-section forces of the cable element and the beam element respectively. Now they can be rewritten:

$$\boldsymbol{N}_c = T\ \boldsymbol{e},\ \ \boldsymbol{N}_b = -T\ \boldsymbol{e} \tag{16}$$

In the result, an applied force acts on a cable in the pulley part directly is considered to be equivalent to pre-stress of cable.

4 NUMERICAL EXAMPLE

4.1 Cable truss system

Analyze a cable truss composed by two main cables and one diagonal cable as shown in Figure 5. The horizontal distance and the height of the cable truss is 40m and 10m respectively. This figure shows an initial shape of cable truss system in non-stress state until an applied load acts on the cable system.

Young's modulus of both kinds of cable is E=140MN/m^2, and cross sectional area of main cable and diagonal cable is 758mm^2 and 39.4mm^2 respectively.

To calculate the equilibrium condition includes cable axial force and cable length of the cable truss system under the action of pre-stress T and own weight q as well as concentrated load P, we need to do the following procedures.

First of form finding procedure, pulleys are installed at all nodes except node 1 and node 7. The cable pre-stress is given in order to ensure that these pulleys do not move in the horizontal direction even if configuration of cable is deformed.

The cable truss system is acted by an initial tension force as cable pre-stress of T=0.5kN and T=10kN at the pulley part of node 6 and node 11 where both cables passing through. Under the action of given pre-stress T on each cable, the slide displacement is considered to occur inside each pulley, and equilibrium condition is required to find under this state.

In the next step, equilibrium condition is required to find when the cable truss system is loaded by own weight q (63.0N/m in main cables and 3.27N/m in diagonal cables). In this state, the form of the cable truss system is settled under the action of pre-stress T and own weight q.

The slide displacements of all pulleys are restrained after settling the form of the cable truss system under the action of the pre-stress T and own weight q. Additionally, equilibrium state of the cable truss system changed when the concentrated load P acts on cable at node 8.

The geometric configuration and axial forces of cable truss are listed in Tables 1 and 2.

4.2 Suspension bridge

4.2.1 Model

A suspension bridge consists of four main structural elements: main cable, hangers, towers and deck. It is assumed that all hangers are vertically installed so that loads applied to the deck are transmitted to the main cable as vertical concentrated loads under dead loads. This paper describes a method to determine form finding for cable-supported structures based on introducing some constraint conditions of cable pre-stress.

First, known horizontal component of cable tension is assumed to be equal at all pulleys. Secondly, the displacement is supposed to be equal zero at the position in which the deck is supported by cable. Above-mentioned conditions are reasonable because it is considered bending moment of tower is considered no happen under the action of the dead load of deck and bending moment of the deck has the same features as bending moment of continuous beam.

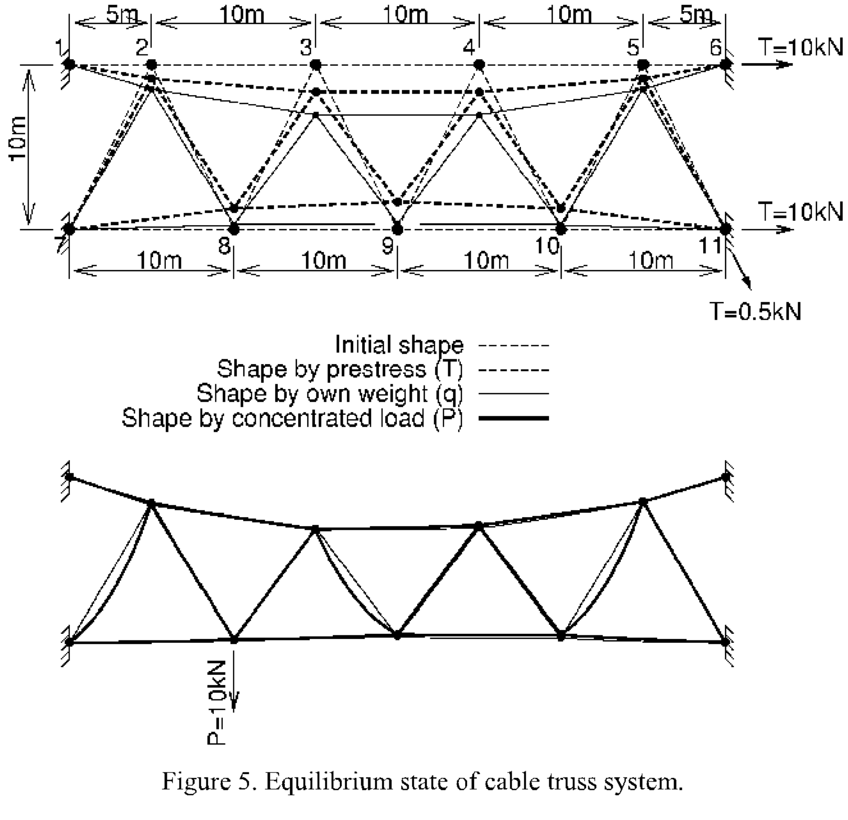

Figure 5. Equilibrium state of cable truss system.

Table 1. Nodal coordinates of the cable truss system

Node	Initial shape		By pre-stress		By own weight		By load P	
	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)
1	0.0000	10.0000	0.0000	10.0000	0.0000	10.0000	0.0000	10.0000
2	5.0000	10.0000	5.0000	9.1529	5.0000	8.4908	4.9579	8.3534
3	15.0000	10.0000	15.0000	8.3301	10.0000	6.9385	14.9692	6.8526
4	25.0000	10.0000	25.0000	8.3301	15.0000	6.9385	24.9699	7.1013
5	35.0000	10.0000	35.0000	9.5129	35.0000	8.4908	34.9667	8.4976
6	40.0000	10.0000	40.0000	10.0000	40.0000	10.0000	40.0000	10.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	10.0000	0.0000	10.0000	1.2411	10.0000	0.2685	10.0048	0.1463
9	20.0000	0.0000	20.0000	1.6454	20.0000	0.3480	20.0018	1.4839
10	30.0000	0.0000	30.0000	1.2411	30.0000	0.2685	30.0043	0.4541
11	40.0000	0.0000	40.0000	0.0000	40.0000	0.0000	40.0000	0.0000

Table 2. Axial forces at center of element and cable lengths of cable truss system

Element	Axial force (kN)			Cable length (m)		
	By pre-stress	By own weight	By load P	By pre-stress	By own weight	By load P
1-2	10.0000	9.9512	33.8842	5.0708	5.2225	5.2225
2-3	9.9653	9.6343	28.8901	10.0329	10.1207	10.1207
3-4	9.9208	9.5177	31.8164	9.9991	10.0009	10.0009
4-5	9.9653	9.6343	32.0038	10.0329	10.1207	10.1207
5-6	10.0000	9.9512	30.7245	5.0708	5.2225	5.2225
				40.2064	40.6873	40.6873
7-8	10.0000	10.0035	20.9467	10.0758	10.0043	10.0043
8-9	9.9109	9.9605	21.3062	10.0072	10.0010	10.0010
9-10	9.9109	9.9605	20.2467	10.0072	10.0010	10.0010
10-11	10.0000	10.0035	21.4261	10.0758	10.0043	10.0043
				40.1660	40.0107	40.0107
7-2	0.5000	0.5138	0.0303	10.4286	9.8531	9.8531
2-8	0.5000	0.5143	6.8735	9.3584	9.6228	9.6228
8-3	0.5000	0.5117	5.4640	8.6741	8.3356	8.3356
3-9	0.5000	0.5118	0.0261	8.3470	8.2722	8.2722
9-4	0.5000	0.5118	1.7702	8.3470	8.2722	8.2722
4-10	0.5000	0.5117	1.9402	8.6741	8.3356	8.3356
10-5	0.5000	0.5143	0.0286	9.3584	9.6228	9.6228
5-11	0.5000	0.5138	4.5044	10.4286	9.8531	9.8531
				73.6162	72.1674	72.1674

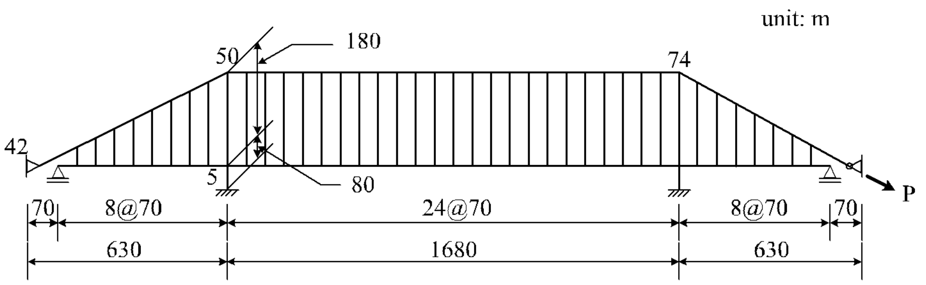

Figure 6. Geometry of suspension bridge.

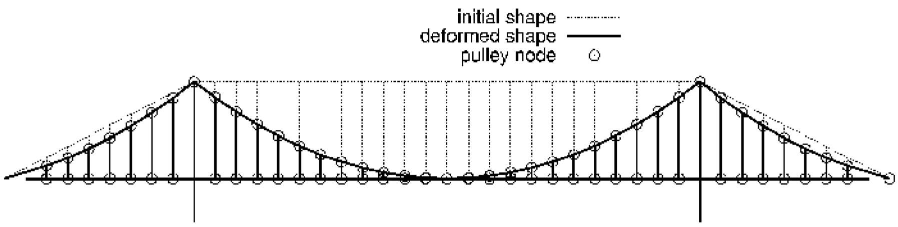

Figure 7. Deformed forme of suspension bridge.

Table 3. Material and sectional properties of the suspension bridge.

Member	E (GPa)	A (m2)	I (m4)	w (KN/m)
Main span cable	210	0.40	-	32.9
Side span cable	210	0.41	-	33.9
Hanger	210	0.025	-	-
Deck	210	0.5	1.66	72.4
Tower (0-80 m)	40	37.5	750	882.0
Tower (80-140 m)	40	32.5	275	764.4
Tower (140-200 m)	40	25.0	200	705.6
Tower (200-260 m)	40	25.0	200	705.6

An earth-anchored suspension bridge likes Figure 6 is selected as the second example. The lengths of the main and side spans are 1680 and 560 m, respectively. The material and sectional properties of main cables, hangers, deck and towers are given in Table 3.

The target profile of the deck is straight without camber. The horizontal positions of the pylon tops are selected so that no bending moment is induced under dead load. The main cable is modeled with 40 cable elements (8 in each side span and 24 in the main span). The deck and the towers are modeled with 280 Euler beam elements and 26 Euler beam elements, respectively. The boundary condition of the end of the deck is described a roller and the boundary condition of the base part of the pylon is described a hinge. Marks ● and ○ are illustrated in Figure 7 denote ordinary node and pulley node, respectively.

Supported condition at node 42 is fixed support. Install a pulley at node 82 so that a large initial tension of the main cable is given to settle configuration of the main cable.

Some pulleys are installed in nodes of the main cable at the position of hangers to express the first constraint condition as shown in Figure 8. The cable pre-stress is given in order to ensure that pulleys are not moved in the horizontal direction.

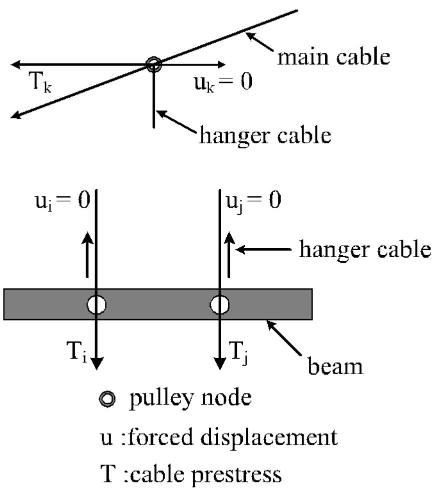

Figure 8. Cable pre-stress is determined basing on known forced displacement.

In addition, to add the second constrain condition which the horizontal component of cable axial force is supposed becomes equal by installing pulley at node 50 and node 74 where the main cable and the pylons are connected as shown in Figure 9.

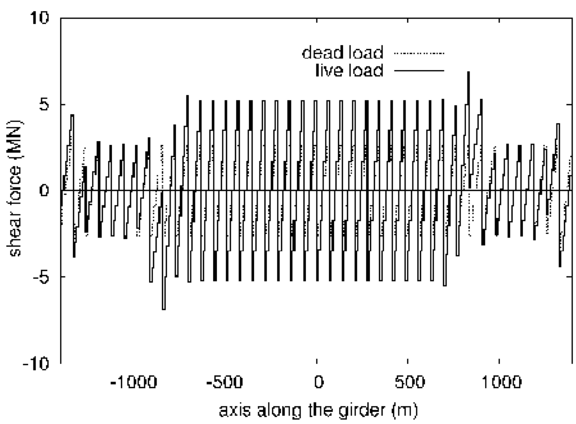

Figure 10. Shear diagram.

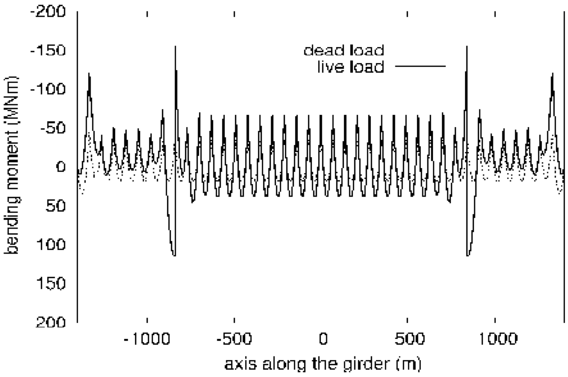

Figure 11. Bending moment diagram.

4.3 Analysis result

At the beginning of numerical process, to settle configuration of main cable, large initial tension of 2000kN is given to act on the main cable in the pulley part of node 82.

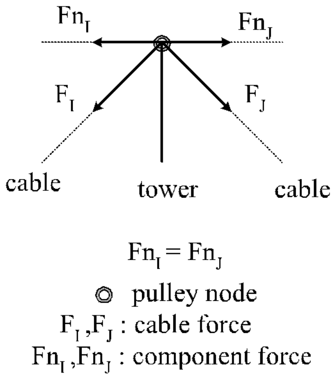

Figure 9. Component of cable axial force at pulley.

Furthermore, the whole span is loaded fully by uniformly distributed load as dead load of the deck. At the result, it is found that pulleys at nodal position where hangers and main cable intersect with, were moved only in the vertical direction. Additionally, it is also found that the vertical displacement of the deck at hanger position does not occur. This is explained that forced displacement was given to each pulley to try to hang the deck from the hanger.

By introducing proposed shape finding method in this research, it is clear to find that distribution of the cross-section force becomes equalization.

According to the result, by substituting ordinary node with pulley, the whole structural system is to be well-balanced. It is also got the length of main cable as well as the cross-section force. It can be said that the shape finding for suspension bridge could be carried out.

After determining the configuration of suspended structure under the pre-stress and the dead load, restrain all installed pulleys, we can consider the analysis result under the action of live load on main span of the deck as Figures 10 and 11.

5 CONCLUSION

It got the following conclusion by this research.

By assuming some conditions that known horizontal component of unit vector of cable axial force is assumed to be equal at all pulleys and the displacement is supposed to be equal zero at the position in which the deck is supported by cable, the problem of form finding for cable-supported structure can be solved. We can understand this content well by consideration of numerical examples.

In this study, by using the proposed pulley-cable element, even if a special calculation technique for finding form is not used, the form finding problem can be solved. That means form finding method and structure analysis method could be done with the same technique.

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